Minh Nguyen

CS 225

4.3 Recursive Definitions

7th edition 5.3: {2(a,b), 8, 24, 26a, 28a, 32a}

2 ) a) f(n+1) = -2f(n)

f(1) = -2f(0) = -2\*3 = -6

f(2) = -2f(1) = -2\*-6 = 12

f(3) = -2f(2) = -2\*12 = -24

f(4) = -2f(3) = -2\*-24 = 48

f(5) = -2f(4) = -2\*48 = --96

b) f(n+1) = 3f(n) + 7

f(1) = 3f(0) + 7 = 3\*3 + 7 = 16

f(2) = 3f(1) + 7 = 3\*16 + 7 = 55

f(3) = 3f(2) + 7 = 3\*55 + 7 = 172

f(4) = 3f(3) + 7 = 3\*172 + 7 = 523

f(5) = 3f(4) + 7 = 3\*523 + 7 = 1576

8) a) an = 4n - 2

base case: a(1) = (4(1)-2 = 2

Recursive step: an = 4(n+1) -2

= 4n +4 - 2

= 4n - 2 + 4

= an + 4

b) an= 1+(-1)n

base case: a1 = 1+(-1)1 = 2

a(n+1) = a(n) + ?

? = an+1 - an

= 1+(-1)n+1 - 1+(-1)n

= -(1)n+1 -1n

= -2

a(n+1)\_ = an - 2

c) an = n(n+1)

base case: a1 = 1(1+1) = 2

an+1 = an + ?

? = an+1 - an

= (n+1)(n+2) - n(n+1)

= n2 + 3n + 2 - n2 - n

? = 2n + 2

= an + 2n + 2

d) an = n2

base case: a1 = 12 = 1

an+1 = an + ?

? = (n+1)2 - n2

= n2 + 2n + 1 - n2

?= 2n + 1

= an + 2n + 1

24) a) Set of odd integers

Basis step: 1 ∈ S

Recursive step: if x ∈ S, then x+2 ∈ S

b) Set of positive integer power of 3

basis step: 3 ∈ S

Recursive step: if x ∈ S, then 3x ∈ S

c) Set of polynomials with integer coefficients

basis step: 0 ∈ S

Recursive step: A(n) ∈ S, then A(n) + B(n)k ∈ S, B ∈ Z, k ∈ Z

Z is the set of integers.

26) a)

1: (2,3), (3,2)

2: (4,6), (5,5), (6,4)

3: (6,9), (7,8), (8, 7), (9, 6)

4: (8, 12), (9,11), ( 10,10) (11,9), (12,8)

5: (10, 15), (11, 14),(12,13),(13,12),(14,11),(15,10)

28 a)

basis step: (0,0) ∈ S

Recursive step: if (a,b) ∈ S, then (a, b+2) ∈ S, (a+2, b) ∈ S, (a+1, b+1) ∈ S

32a) Basis step: ones(λ) = 0, λ is the empty string

Recursive step = if x ∈ S, and w ∈ Σ\*, then ones(wx) = ones(w) + x